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A NETWORK TRANSSHIPMENT MODEL FOR MANPOWER PLANNING AND DESIGN.(U)
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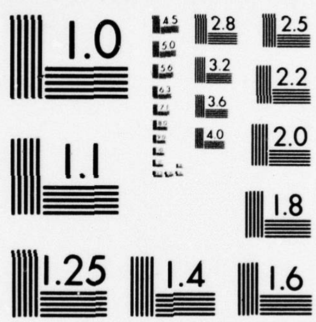

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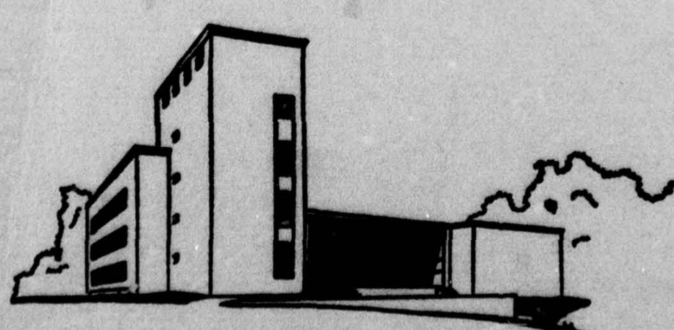

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LEVEL II

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FOR MANPOWER PLANNING AND DESIGN.

by

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A Network Transshipment Model
for Manpower Planning and Design

by

Gerald L. Thompson
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1. INTRODUCTION

In [5] R. C. Grinold proposed a linear programming cohort model which could be used for studying the design of a manpower system. In the present paper it is suggested that his linear programming model can be approximated by a network transshipment model, in the same way that A. Charnes, W. W. Cooper, K. A. Lewis, and R. J. Niehaus suggest approximating a certain goal programming model with an imbedded Markov chain by a network model, see [1,2]. The loss of generality in such approximations is slight, while there are substantial gains possible in terms of computing speeds as shown by F. Glover, K. Karney, and D. Klingman in [4] and V. Srinivasan and G. Thompson in [10].

Besides the increase in speed of computation, many other advantages become evident. One is the availability of the operator theory of parametric programming for such problems developed by Srinivasan and Thompson in [9] which makes the process of approximating a linear program by a network model not difficult. It also makes the computation of the pareto optimal effectiveness-cost tradeoff curve discussed in Section 5 very easy, see [11].

In Section 2 the original model by Grinold is briefly described, and then translated into a network transshipment model. Two objective functions, minimum cost and maximum effectiveness are considered. In Section 3 a simple numerical example with minimum cost objective is solved. Section 4 discusses the interpretations of the dual matrix for the example and for the maximum

effectiveness objective. In Section 5 the model is considered with the two objectives simultaneously evaluated, and the problem of finding the effectiveness-cost tradeoff curve is solved. Section 6 is devoted to career analysis that is available from the outputs to the model, and Section 7 gives conclusions.

It is with the greatest pleasure that I dedicate this paper to Professor William W. Cooper. He was the one who first introduced me to the manpower planning area and who has consistently supported my work as well as that of many others in the area. His unselfish devotion to scientific research, of all kinds, has greatly enriched every area which he has touched.

2. THE MODEL

We begin by introducing some notation very similar to that employed by Grinold in [5]. After a brief discussion of the problem in that notation we will change notation to make possible a network formulation.

Consider the manpower system which has ranks $r \in \{1,2,\ldots,R\}$, with increasing rank associated with increasing rank number, and years of service $t \in \{1,2,\ldots,T\}$. Thus T is the maximum period of service and retirement must take place on or before the beginning of year $T + 1$. We define the following quantities:

(r,t) = state of being in rank r serving in year t.

K = number of states (r,t) for $1 \leq r \leq R$ and $1 \leq t \leq T$.

$n(r,t)$ = number of people in rank r serving in year t.

There are two terminal states, Retired and Separated.

$f[(r,t);(r,t+1)]$ = fraction of people in state (r,t) going to state $(r,t+1)$

$f[(r,t);(r+1,t+1)]$ = fraction of people in state (r,t) being promoted at time t to state $(r+1,t+1)$

$1-f[(r,t);(r,t+1)] - f[(r,t);(r+1,t+1)]$ = fraction of people in state (r,t) being separated from service at time t

$c[(r,t)]$ = cost of having a person serve his year t in rank r

$e[(r,t)]$ = effectiveness or value of having a person serve his year t in rank r.

We call the above description of the model the state model. Figure 1 shows the possible states and transitions among them for a state model in which $R = 3$ and $T = 5$. After making the assumption that effectiveness as well as costs are additive, Grinold goes on to state the problem of optimal manpower system design as a linear programming problem of either(a) maximizing total effectiveness subject to cost, time in rank, minimum force size, average length of service, and other constraints; or (b) minimizing total cost subject to minimum effectiveness, and other constraints similar to the above. He goes on to suggest that sensitivity analysis of the linear program would supply valuable information.

In [2,3] Charnes, Cooper, Lewis, and Niehaus show how a linear programming manpower system model can be approximated by means of capacitated network model. Here we shall adapt their suggestion to Grinold's model.

The first step is to relabel the states using a lexicographic ordering. Figure 2 shows the new and old listing of states for the example of Figure 1. Note that the retirement and separation states are numbered $K+1$ and $K+2$, respectively.

The network with the new state labels is shown in Figure 3. For the time being ignore the numerical example which is also imposed on that figure.

We now introduce the notation necessary for the transshipment model. First the "from" states

$$I = \{1,2,\ldots,K\};$$

then the "to" states

$$J = \{2,\ldots,K,K+1,K+2\}\ .$$

We call state 1 the <u>injection state</u>; states 2 through K are called <u>transshipment states</u> because they appear both as "from" and "to" states; finally, state $K+1$ is the <u>retirement state</u> and state $K+2$ is the <u>separation state</u>. Next we define

$$x_{ij} = \underline{\text{manpower flow}} \text{ from state } i \in I \text{ to state } j \in J$$

These variables are bounded by

$$U_{ij} = \begin{cases} 0 \text{ if manpower cannot flow from state } i \text{ to } j \\ \text{upper bound on the flow when flow is possible.} \end{cases}$$

These upper bounds are varied to achieve the desired fractions of people being promoted, continued in rank, separated, etc. of the state model. We also define initial and final flows

x_0 = yearly injection of new employees into state i

x_R = yearly employee retirements

x_S = yearly employee separations.

For feasibility we assume

$$x_0 = x_R + x_S \tag{1}$$

With the above notation we can now state the feasibility constraints for the transshipment model:

$$\sum_{j \in J} x_{ij} = x_0 \quad \text{for all } i \in I \tag{2}$$

$$\sum_{i \in I} x_{ij} = x_0 \quad \text{for all } j \in J-\{K+1, K+2\} \tag{3}$$

$$\sum_{i \in I} x_{i,K+1} = x_R \tag{4}$$

$$\sum_{i \in I} x_{i,K+2} = x_S \tag{5}$$

$$0 \leq x_{ij} \leq U_{ij} \quad \text{for all } i \in I,\ j \in J. \tag{6}$$

If i is a transshipment state, that is, $i \in I \cap J = \{2,\ldots,K\}$. then the variables x_{ii} are called transshipment variables. If index i corresponds to state (r,t) the transshipment variables have the following special interpretations:

$x_0 - x_{ii}$ = the number of employees in rank r at time t

x_{ii} = the number of employees not in rank r at time t.

Obviously employees not in rank r at time t are either in other ranks, or have retired, or have been separated.

We turn now to the statement of objective functions for the model. We shall consider here just two objectives, minimize cost, or maximize effectiveness. For the cost objective we define

$$c_{ii} = 0 \quad \text{for } i \in I \cap J \tag{7}$$

$$c_{ij} \begin{cases} = \infty & \text{if state } i \text{ is not connected by an arc to state } j\text{, or} \\ = & \text{yearly cost of having an employee transiting from } i \in I \text{ to } j \in J \text{ where } i \neq j \end{cases} \tag{8}$$

$$c_{i,K+1} \begin{cases} = \infty & \text{if retirement is impossible from } i\text{, or} \\ = & \text{total retirement cost if retirement is possible from } i \end{cases} \tag{9}$$

$$c_{i,K+2} = \text{cost of separating an employee from state } i \tag{10}$$

With these definitions the minimum cost transshipment objective function is

$$\text{Minimize } \{C = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}\}. \tag{11}$$

The problem stated in (2)-(6) and (11) is a <u>transportation problem</u>, which is a linear programming problem having special structure. The dual problem is given by

$$\text{Maximize } \sum_{i \in I} a_i u_i + \sum_{j \in J} b_j v_j \tag{12}$$

Subject to

$$u_i + v_j - w_{ij} \leq c_{ij} \quad \text{for } i \in I \text{ and } j \in J \tag{13}$$

$$w_{ij} \geq 0 \quad \text{for } i \in I \text{ and } j \in J \tag{14}$$

where the dual variables u_i for $i \in I$ are associated with constraints (2), the variables v_j for $j \in J$ are associated with constraints (3), (4), and (5), and the variables w_{ij} for $i \in I$ and $j \in J$ are associated with constraints (6).

For a given basis B it is possible to determine a one-parameter family of solutions u_i and v_j to the equations

$$u_i + v_j = c_{ij} \quad \text{for} \quad (i,j) \in B \tag{15}$$

such that $d_{ij} = u_i + v_j$ is unique. The matrix $D = \{d_{ij}\}$ the <u>dual matrix</u> of the dual solution given by a basis B. As noted in [9] it can easily be shown that

$$w_{ij} = \max(0, u_i + v_j - c_{ij}) \text{ for } i \in I \text{ and } j \in J \tag{16}$$

for any dual feasible basis B. Therefore the dual variables u_i and v_j suffice to characterize the dual solution.

For the effectiveness objective we define an effectiveness function e_{ij} on arcs (i,j) as follows:

$$e_{ii} = 0 \quad \text{for} \quad i \in \cap J \tag{17}$$

$$e_{ii} \begin{cases} = -\infty \text{ if state } i \text{ is not connected by an arc to state } j, \text{ or} \\ = \text{ yearly effectiveness of having an employee} \\ \quad \text{transiting from state } i \in I \text{ to } j \in J \end{cases} \tag{18}$$

$$e_{i,K+1} = 0 \quad \text{for} \quad i \in I \tag{19}$$

$$e_{i,K+2} = 0 \quad \text{for} \quad i \in I \quad . \tag{20}$$

The problem of determining the effectiveness of <u>single</u> individuals has been addressed by using Delphi techniques by J. R. Schmidt and R. K. Hovey in [7]. They had difficulty extending these individual results to effectiveness of groups of individuals. Nevertheless, we shall here make the (admittedly heroic) assumption that effectiveness is an additive function. Thus the total effectiveness objective is

$$\text{Maximize} \quad \{E = \sum_{i \in I} \sum_{j \in J} e_{ij} x_{ij}\} . \tag{21}$$

It should be remarked that in [5] Grinold treats a case in which the total effectiveness function is non-linear. As might be expected, the computational problems in that case are difficult.

The dual problem to the transportation problem defined by (2)-(6) and (21) is just like the one given by (12)-(14) except that the word "Maximize" in (12) must be replaced by the word "Minimize." The same remarks made previously about the dual problem apply here.

3. AN EXAMPLE

A numerical example with cost minimization objective function is shown in network form in Figure 3 and in transshipment form in Figure 4. Actual numbers were chosen for c_{ij} and U_{ij} for each arc; also $x_0 = 1000$, $x_R = 681$ and $x_S = 319$ were chosen. From the actual flows in the solution to the problem it is easy to calculate the fractions of people retained in rank $f[(r,t);(r,t+1)]$, the fractions of people being promoted $f[(r,t);(r+1,t+1)]$, and from these two the fractions of people being separated at each state. The upper bounds U_{ij} must be selected by trial and error to (a) achieve feasibility, and (b) to achieve approximately the desired promotion fractions in-rank fractions at each state. Charnes, Cooper, Lewis, and Niehaus selected upper bounds to achieve similar fractions in their models [2,3].

It is obvious that with the minimum cost objective, the optimal solution will try to make the following decisions on the arcs leading from a given state i:

(a) Separate as many persons as possible; i.e., put maximum flow possible on the upward slanting arrows in Figure 3;

(b) Retain as many people in rank rather than promote them; i.e., put maximum flow possible (after satisfying (a)) on the horizonal arrows in Figure 3. (This remark does not hold on the bottom row of the figure where no promotion is possible.)

From these remarks it follows that the basis cells of the optimal solution to the transshipment problem in Figure 3 will consist of (i) the transshipment cells and (ii) the promotion cells, plus other cells as necessary to complete the basis. The other cells include the in-rank cells at the highest rank, since promotion is not possible from these states, and the retirement cells in column 13, since again no other alternative is available. (There is one additional cell (5,14) needed to complete the basis.) These conclusions can be confirmed by looking at the circled basis cells in Figure 4 and the upper bounded cells in that figure indicated by squares.

From these remarks it might and can be concluded that the solution to the transportation problem, or the network problem of Figure 3 is not difficult. This is true, and advantage can be taken of the fact that good starting bases are easy to find, in the development of special algorithms for solving the problem.

With the maximum effectiveness objective function, the corresponding obvious optimal decisions are:

(c) Promote as many people as possible; i.e., put the maximum flow possible on the downward slanting arrows in Figure 3;

(d) Retain as many people in rank as possible rather than fire them; i.e., put maximum flow possible (after satisfying (c)) on the horizontal arrows.

From these remarks most of the optimum basis cells can be identified in advance for this case as was done previously for the minimum cost objective case.

4. DUAL VARIABLE ANALYSIS

The optimal dual matrix for the problem in Figure 4 is shown in Figure 5. We use the results of Srinivasan and Thompson [9] to interpret the relevant entries of the dual matrix. Although every entry in this matrix has one or more interpretations, only the entries in the column 13 corresponding to retirement and the entries in other rows that have squares around them corresponding to cells at their upper bounds, are of interest here. Also it turns out that the optimal basis subgraph of the network of Figure 1 is needed to make such interpretations. This basis subgraph is shown in Figure 6. Notice that none of the horizontal arcs (except those in the bottom row) are in the basis graph; all these arcs, which correspond to "retain in rank" are used to the fullest capacity, i.e., are upper bounded, and hence cannot be in the basis graph. Similarly all the upward slanting arcs, which correspond to separation, except for the one from node 5, are upperbounded and hence not in the basis graph. Because all the arcs corresponding to promotions are in the basis this can be called the <u>most expensive basis graph.</u> The reason that the optimum solution to the cost minimizing problem leads to the most expensive basis graph is, of course, that all the cheaper arcs are already utilized to their fullest, i.e., to their upper bounded capacity, in the solution. Hence only the expensive arcs involving promotions are available to be in the basis. There is one exception to this rule, namely the separation arc from node 5. But that arc has to be in the basis to make the basis graph be connected, i.e., touch all the nodes. It is also not upper bounded in order that the solution can achieve the desired number (319) of separations.

We can now state a general rule for interpreting the numbers in the dual matrix of Figure 5:

Dual Matrix Rule. Construct the directed basis graph G of an optimal solution to the transportation problem; find the unique path in G from node i to node j; then the entry d_{ij} of the dual matrix is the directed cost of the path from i to j, i.e., it is the sum of the signed arc costs on the path with a plus sign on the cost when the arrow on the arc points in the same direction as the path, and a minus sign on the cost when the arrow points in the opposite direction as the path.

For instance, consider the 15 entry in the upper left hand corner of Figure 5; it corresponds to $i = 1$ and $j = 2$. In Figure 6 the path from node 1 to node 2 consists of nodes 1, 6, 10, 11, 7, and 2 in that order. The sum of the signed cost entries on this path is

$$12 + 16 + 17 - 17 - 13 = 15$$

as indicated by the dual variable rule. This same path is indicated by the nodes connected by arcs in Figure 4 (the transshipment nodes (i,i) should be ignored, since the cost on such nodes is 0.)

As a second example, consider the entry 223 in row 1 and column 13 of Figure 5. Applying the dual variable rule we see that it should be the sum of the costs on the path from node 1 to node 13 in Figure 6. Adding this gives

$$12 + 16 + 17 + 18 + 160 = 223$$

which verifies the rule. In the same way it is easy to check the following: The entries in column 13 give the most expensive retirement paths from the states corresponding to rows. In other words, these numbers give upper bounds on the total cost of having an employee in each of the employment states.

In order to interpret the entries of the dual matrix marked by squares in Figure 5, which correspond to upper bounded cells, we make use of the results of Srinivasan and Thompson [9] on bound operators for parametric programming for the transportation problem. Specifically we make use of the definition of a bound operator on p. 221 and Theorems 9 and 10 on p. 223 of that reference. The results given there can be briefly summarized as follows: Suppose at cell (p,q) the upper bound U_{pq} is changed to

$$U'_{pq} = U_{pq} + \delta \tag{22}$$

where δ is a positive or negative number that is "sufficiently small" in a sense to be defined shortly, see (25) below. Let B be the basis of an optimal solution to the original problem. Then there exist positive constants μ^{+}_{pq} and μ^{-}_{pq} such that, if δ satisfies

$$-\mu^{-}_{pq} \leq \delta \leq \mu^{+}_{pq} \tag{23}$$

then B is still the optimal basis to the new problem which is the same except U_{pq} is replaced by U'_{pq}. Moreover, if Z and Z′ are the original and new objective function values then

$$Z' = Z \quad \text{if} \quad (p,q) \notin UB \tag{24}$$

$$Z' = Z - \delta d_{pq} \quad \text{if} \quad (p,q) \in UB \tag{25}$$

where UB is the set of cells for which $x_{pq} = U_{pq}$, i.e., cells at their upper bounds. The determination of the numbers μ^{+}_{pq} and μ^{-}_{pq} is not difficult and is explained in reference [9].

In essence what (24) says is that if (p,q) is not in the set of upper bounded cells then changes can be made to U_{pq} within limits given by (23)

<u>without</u> changing the optimal solution or objective value of the problem. What (25) says is that if (p,q) is in the set of upper bounded cells, which is true for the cells marked with squares in Figure 5, then making changes of type (22) cause changes of type (25) provided δ satisfies (23). Since the upper bounds U_{pq} can be changed by the decision maker in order to achieve certain goals involving promotion fractions, in-grade constraints, separation constraints, etc., we see that the latter dual variable interpretations are very important. We summarize these in the following rule: <u>For the upper bounded cells (p,q) (marked with squares in Fig. 5), changes of form $U'_{pq} = U_{pq} + \delta$ imply cost changes of form $Z' = Z - \delta d_{pq}$ provided δ satisfies the constraints</u> $-\mu^-_{pq} \leq \delta \leq \mu^+_{pq}$.

As an example consider the cell (1,2) in Figure 5; here $d_{12} = 15$ and $x_{12} = U_{12} = 850$ as shown in Figure 3. If we now replace U_{12} by $U'_{12} = 850 + \delta$ we see that the objective function cost change is given by $Z' = Z' - 15\delta$. In other words, <u>increasing</u> U_{12} from 850 to 851 by making $\delta = 1$ <u>decreases</u> the objective function cost by 15; whereas <u>decreasing</u> U_{12} from 850 to 849 by making $\delta = -1$, increases the objective function cost by 15. The way the optimal solution is changed in each of these cases can be found by tracing out the cycle shown in Figure 4. It can also be found from the graph of Figure 6 by tracing out the path from node 1 to node 2. We leave the details of the determinations of these paths to the reader because of lack of space.

The dual variable analysis for the maximum effectiveness objective function (21) case proceeds in an exactly similar manner, but, of course, with suitable changes in signs. Here the optimal basis graph will correspond to a "least effective" graph such as the one shown in Figure 7. Details are not given here for lack of space.

5. EFFECTIVENESS VERSUS COST TRADEOFF ANALYSIS

For most purposes neither the minimum cost nor the maximum effectiveness objective function previously considered are necessarily the most appropriate. The usual situation is that a given amount of money is budgeted for personnel purposes, and the objective is to obtain the most effective force that costs the budgeted amount. However, once that problem is solved additional questions are frequently asked such as: Given that a certain level of effectiveness has been achieved with the allocated budget how much more (or less) effectiveness can be obtained with x percent more (or less) money? Or, what will it cost to achieve a given level of effectiveness?

The best way to answer these, and other similar questions is to compute the effectiveness-cost tradeoff curve. For the model being considered this turns out to be relatively easy if we use the method given by Srinivasan and Thompson in [11] for solving multiple objective transportation problems. Consider the problem $P(\delta)$ with constraints given by (2)-(6) and the following objective function

$$\text{Minimize } \{J = C - \delta E\} \tag{26}$$

where C is the cost function of (11), E is the effectiveness function of (21), and δ is a nonnegative parameter. When $\delta = 0$, the problem is the minimum cost problem previously considered. As δ increases the optimum solution to the problem gives a more costly, but also more effective solution. In Figure 8 the effectiveness-cost tradeoff curve is shown. As Srinivasan and Thompson have shown in [ ], this curve is pareto-optimal, is non-decreasing, and is piecewise linear, see also Geoffrion [ ]. In that reference they exhibit a very efficient parametric programming method for computing the

trade-off curve. The reader is referred to that paper for details. As noted in Figure 8, points above the trade-off curve are infeasible, and points below are non-optimal. A curve having these properties is said to be pareto-optimal.

For any given point on the trade-off curve the optimum basis graph will be a directed tree but with a less obvious structure than those exhibited in Figures 6 and 7. Hence it is not as easy as before to guess a good starting solution. However, the dual matrix rule of Section 4 is still valid and can be used to interpret the optimum solution in the same way.

6. CAREER ANALYSIS

The model shown in Figures 1 and 3 can be called a cohort model because it traces throughout their careers the flow of a group of persons all of whom enter the system at the same time. We can turn it into an equilibrium steady state model by considering five such networks each starting at the beginning of five consecutive years and look across all five for a single year. More simply we can just sum across all the years in Figure 3 to find the numbers of persons in each rank. These sums show that of the 5,000 people injected into the employment system over a period of 5 years there are 3046 in active service in equilibrium. They are distributed in the various ranks as indicated in Figure 9. Note that there are 93% in rank 1 positions, 6 percent in rank 2 positions, and 1 percent in rank 3 positions.

In the same way, if we look at the solution in Figure 3 to find the eventual fate of newly entering employees we see from Figure 10 that 32% are separated before retirement, 48 percent retire from rank 1 positions, 17 percent from rank 2 positions, and 3 percent from rank 3 positions. Also shown in Figure 10 are the conditional percentages of employees retiring from

each rank, given that they are not separated before retirement. Note that these percentages are 71 percent from rank 1, 24 percent from rank 2, and 5 percent from rank 3. When compared to the corresponding numbers in Figure 9, these percentages may seem somewhat paradoxical: for instance, although only 1 percent of active employees in any year are in rank 3 positions, we see that 5 percent of those retiring in any year do so from rank 3 positions. Similar comparisons can be made from each of the other ranks. The paradox can easily be explained by considering the fact that retirees are chosen from the oldest segment of current employees and they are more likely to be in the higher ranks.

Still another use of the cohort flow model in Figures 1 and 3 is that of successful career analysis. By a successful career we shall mean one that ends in retirement (rather than separation). The following very simple algorithm can be used to count the number of successful careers.

(a) Mark the retirement state with a 1.

(b) Mark each state, all of whose successors are already marked, with the sum of the marks of its successors.

(c) Stop when the injection state is marked. Its mark gives the number of successful careers that are possible.

The calculations of the algorithm when applied to the network in Figure 1 are given in Figure 11. They show that 11 successful career paths are possible from the injection state to the retirement state. These successful career paths are most easily summarized by indicating the states at which promotions take place, as indicated in Figure 12. The career marked 1 means that the person entered employment at injection node 1 and was never thereafter promoted. Career 1,9 indicates a career starting at 1 going to 4, then to 9, and then to retirement. Etc..

In order to get an idea of how common each of these successful careers is we trace each one in Figure 3 and record the arc along which the fewest people can travel in the career because it has the smallest optimal flow of any arc on the path; such arc capabilities are called bottleneck flows. Each of these bottleneck flows, and its percentage of the sum of all the flows is marked in Figure 12. Although these percentages are not an accurate evaluation of the difficulty of following a given career, they do give some indication of the relative difficulty of a given entrant following each of the 11 successful career paths. In a real employment situation, these percentages could be compared with actual frequencies of the successful career paths. For instance I. P. Sherlock reports in [8] the proportions of people in each rank for certain trades in the Canadian armed forces.

7. CONCLUSIONS

The goal of this paper was to demonstrate the usefulness of a network transshipment model to approximate a linear programming model for manpower planning and design. The main advantages of the network model are: fast computing speeds, easy computation of effectiveness-cost trade-off curve, the availability of much managerial information contained in the dual matrix, and its usefulness in analyzing career paths. The only appreciable disadvantage is the difficulty of imposing extra constraints. In any specific instance, the latter difficulty can usually be overcome by employing parametric programming procedures as illustrated above.

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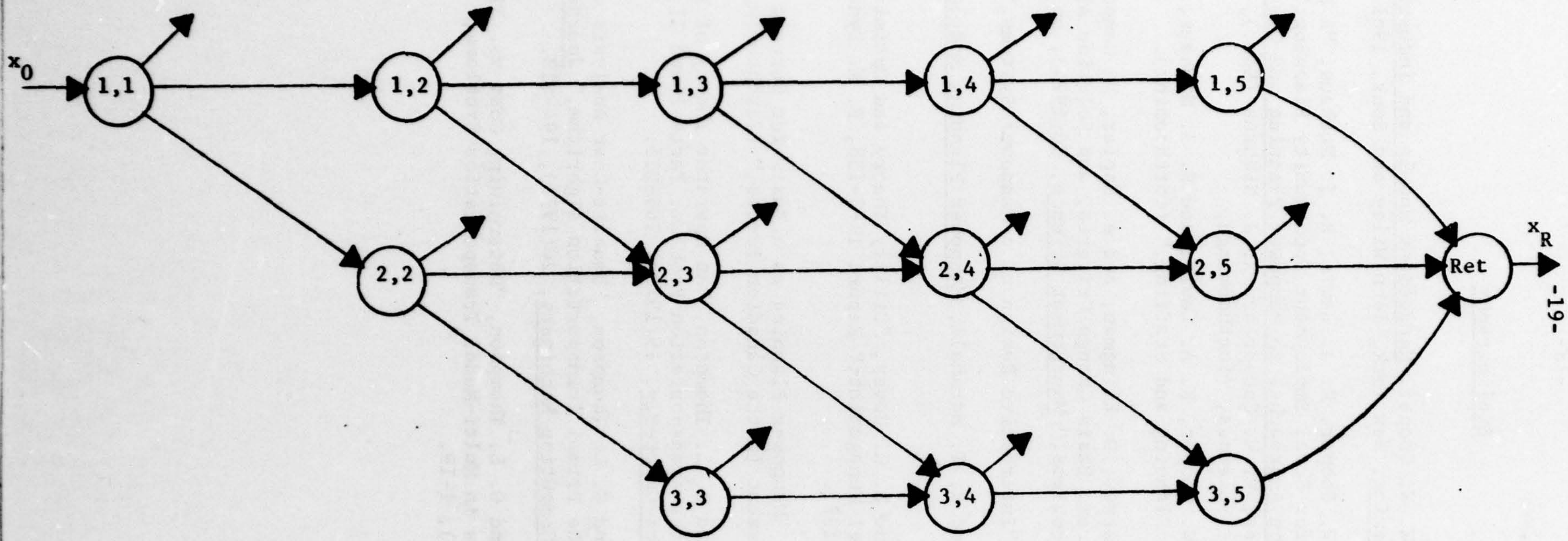

Figure 1. State version of the manpower model with $R = 3$ and $T = 5$. Upward slanting arrows denote separations, horizontal arrows denote continuation in rank, and downward slanting arrows denote promotions. The yearly number of new employees is x_0, the yearly number of retirements is x_R, and the yearly separations (sum of flows on all upward slanting arrows) is x_S; we require $x_R + x_S = x_0$.

Number	State
1	(1,1)
2	(1,2)
3	(1,3)
4	(1,4)
5	(1,5)
6	(2,2)
7	(2,3)
8	(2,4)
9	(2,5)
10	(3,3)
11	(3,4)
K = 12	(3,5)
K + 1 = 13	Retirement
K + 2 = 14	Separation

Figure 2. Lexicographic numbering for the example of Figure 1.

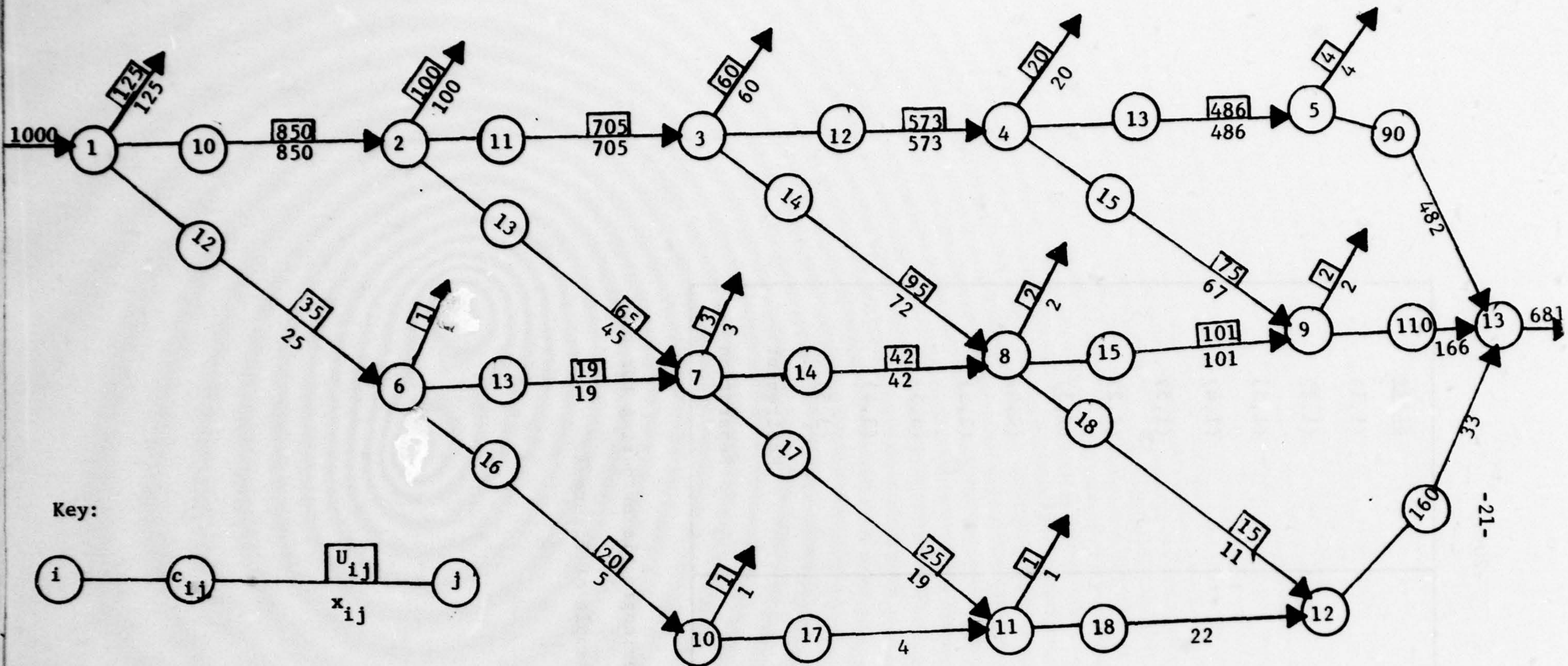

Figure 3. Network Model for example of Figure 1. On each arrow the circled number represents the cost of an employee being in the state labelling the start of the arrow. The number in the box represents the maximum flow between the state at the start to the state at the end of the arrow; the number under the box is the actual flow. Note that $x_0 = 1000$; $x_R = 681$, and $x_S = 319$.

	2	3	4	5	6	7	8	9	10	11	12	13	14	
1	$\boxed{10}$ 850				12 25								$\boxed{0}$ 125	1000
2	0 150	$\boxed{11}$ 705				13 45							$\boxed{0}$ 100	1000
3		0 295	$\boxed{12}$ 573				14 72						$\boxed{0}$ 60	1000
4			0 427	$\boxed{13}$ 486				15 67					$\boxed{0}$ 20	1000
5				0 514								90 482	0 4	1000
6					0 975	$\boxed{13}$ 19			16 5				$\boxed{0}$ 1	1000
7						0 936	$\boxed{14}$ 42			17 19			$\boxed{0}$ 3	1000
8							0 886	$\boxed{15}$ 101			18 11		$\boxed{0}$ 2	1000
9								0 832				110 166	$\boxed{0}$ 2	1000
10									0 995	17 4			$\boxed{0}$ 1	1000
11										0 977	18 22		$\boxed{0}$ 1	1000
12											0 967	160 33		1000
	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	681	319	

Figure 4. Transshipment model for network model of Figure 3

	2	3	4	5	6	7	8	9	10	11	12	13	14
1	$\boxed{15}$	31	98	133	12	28	45	113	28	45	63	223	$\boxed{133}$
2	0	$\boxed{16}$	83	118	-3	13	30	98	13	30	48	208	$\boxed{118}$
3	-16	0	$\boxed{67}$	102	-19	-3	14	82	-3	14	32	192	$\boxed{102}$
4	-83	-67	0	$\boxed{35}$	-86	-70	-53	15	-70	-53	-35	125	$\boxed{35}$
5	-118	-102	-35	0	-121	-105	-88	-20	-105	-88	-70	90	0
6	3	19	86	121	0	$\boxed{16}$	33	101	16	33	51	211	$\boxed{121}$
7	-13	3	70	105	-16	0	$\boxed{17}$	85	0	17	35	195	$\boxed{105}$
8	-30	-14	53	88	-33	-17	0	$\boxed{68}$	-17	0	18	178	$\boxed{88}$
9	-98	-82	-15	20	-101	-85	-68	0	-85	-68	-50	110	$\boxed{20}$
10	-13	3	70	105	-16	0	17	85	0	17	35	195	$\boxed{105}$
11	-30	-14	53	88	-33	-17	0	68	-17	0	18	178	$\boxed{88}$
12	-48	-32	35	70	-51	-35	-18	50	-35	-18	0	160	$\boxed{70}$

Figure 5. Optimal dual matrix for problem in Figure 4

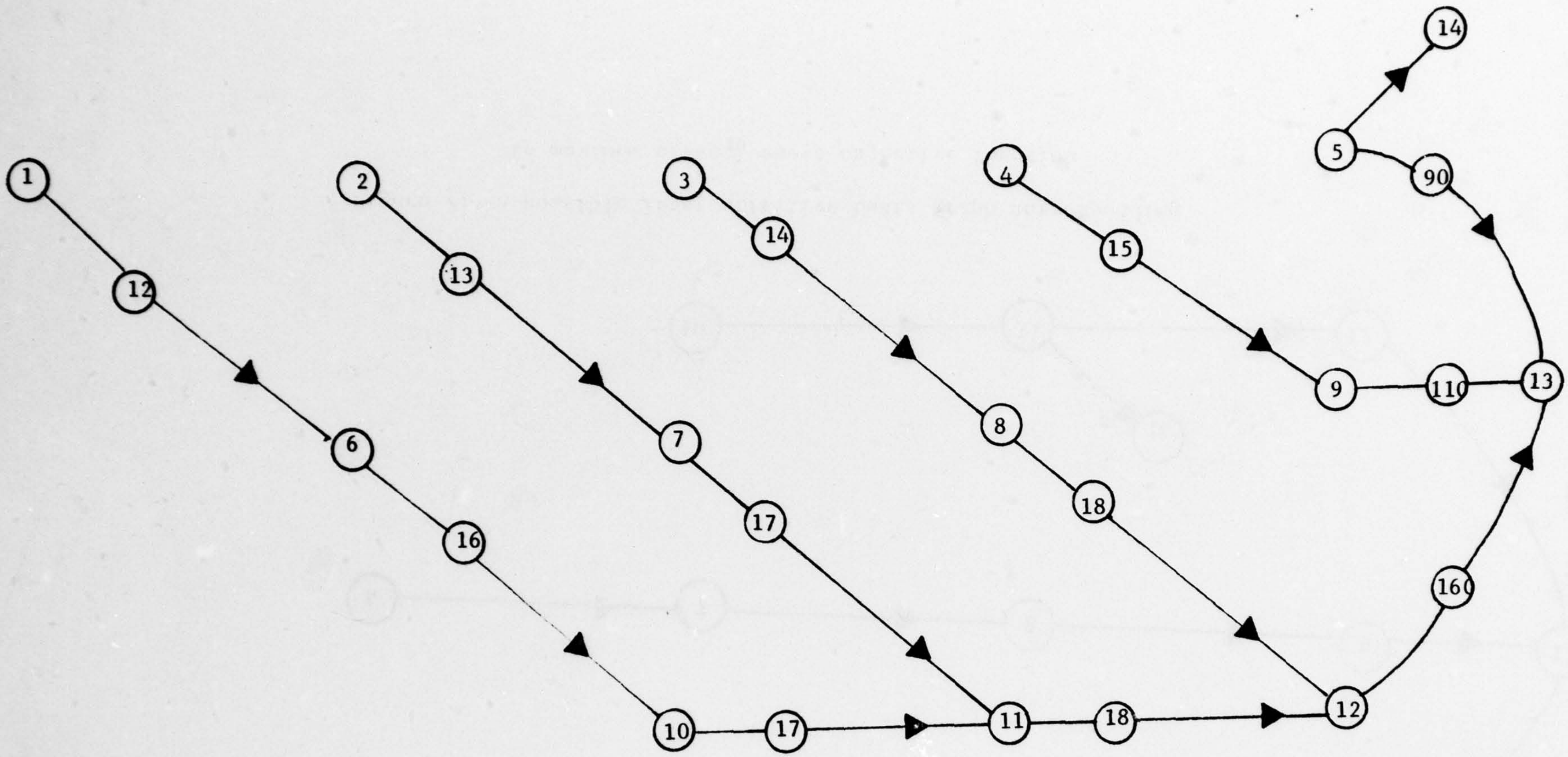

Figure 6. Most expensive basis graph corresponding to minimum cost objective function

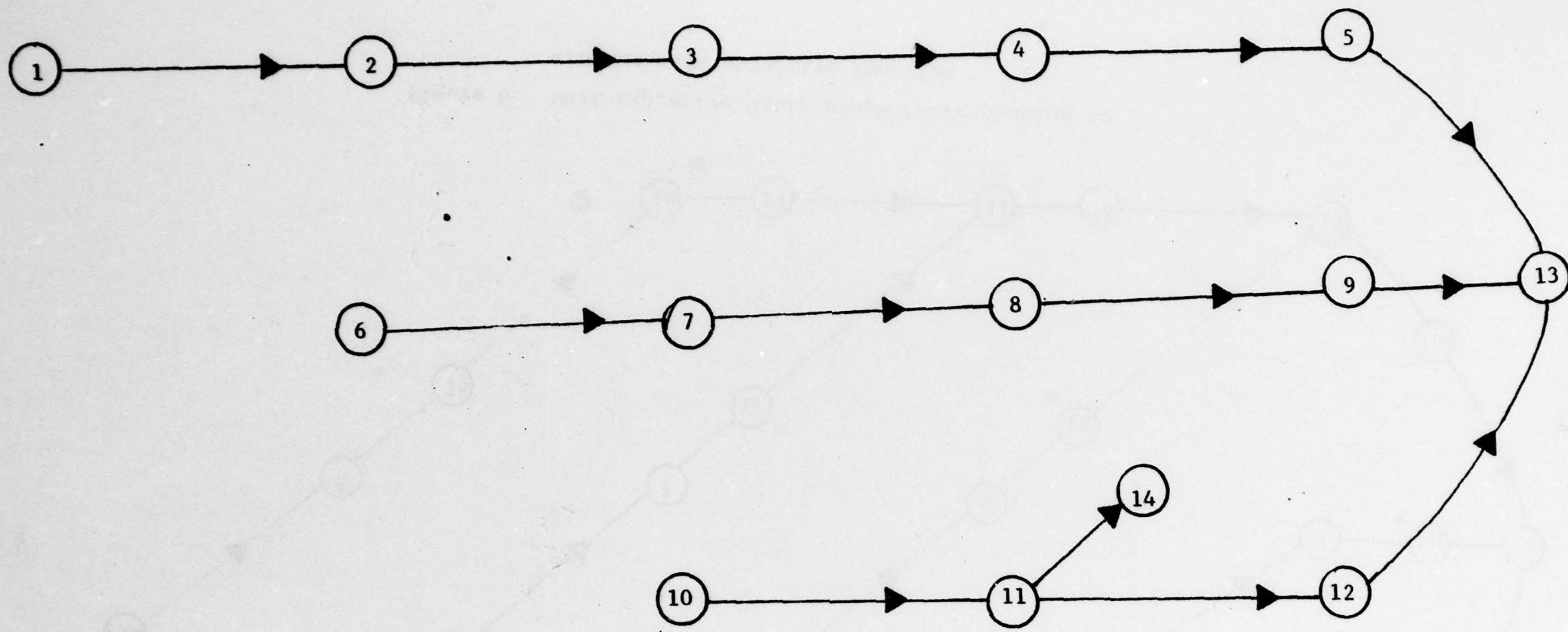

Figure 7. A possible least effective basis graph corresponding to maximum effectiveness objective function

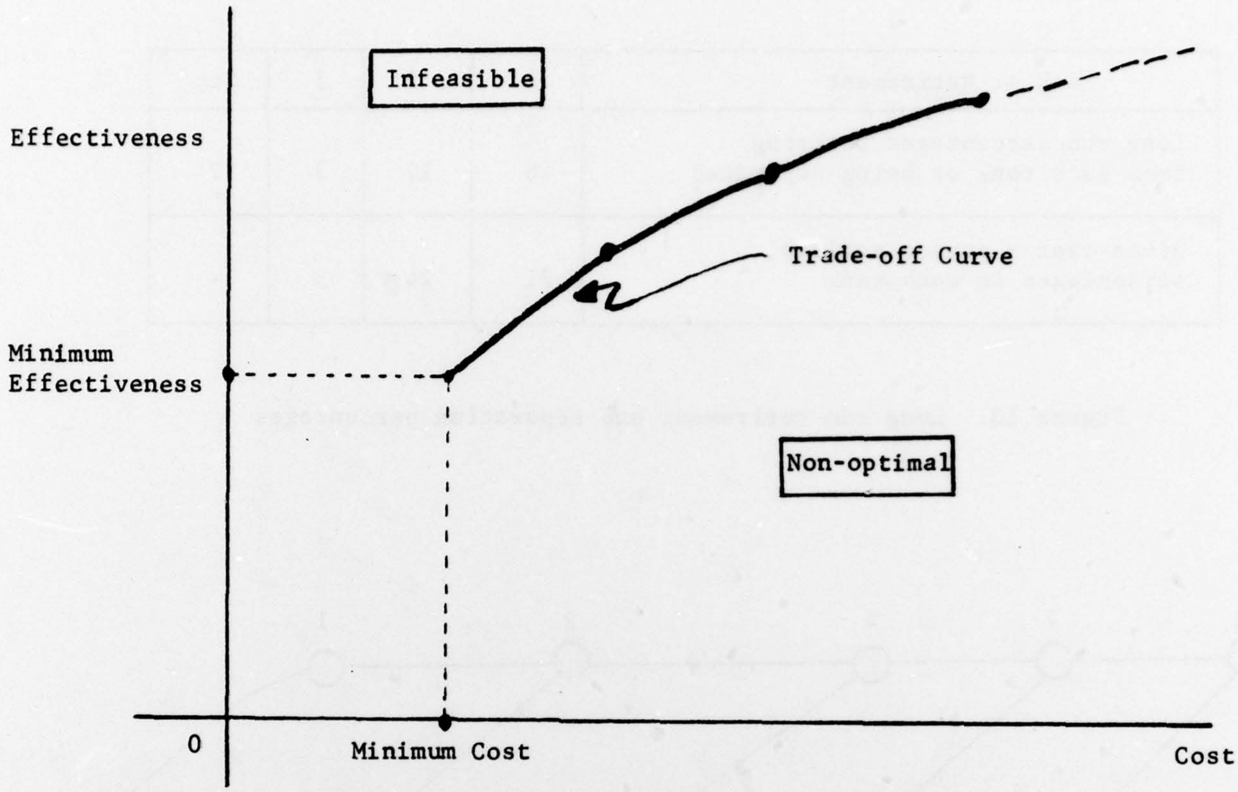

Figure 8. Effectiveness-Cost Trade-off Curve

Ranks	1	2	3
Equilibrium numbers	2823	197	26
Equilibrium percentages	93	6	1

Figure 9. Long run equilibrium numbers and percentages of persons in each rank.

Rank at Retirement	1	2	3	Sep
Long run percentages retiring from each rank or being separated	48	17	3	32
Given that a person retires, percentages in each rank	71	24	5	-

Figure 10. Long run retirement and separation percentages

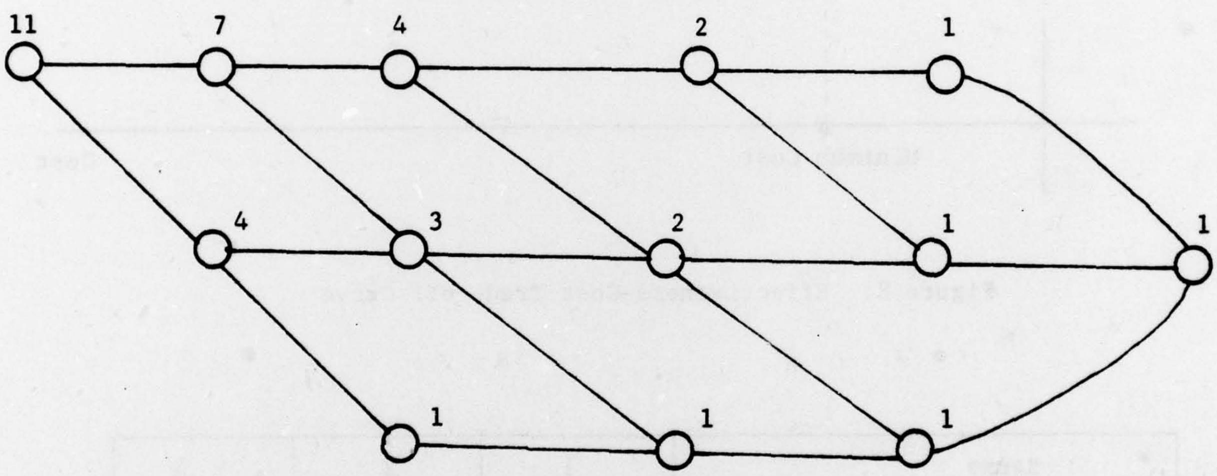

Figure 11. Application of the algorithm for counting the number of successful careers. Note that 11 such careers are possible.

Career Paths	Bottleneck Flows	Percentages
1	482	63
1,9	67	9
1,8	72	10
1,7	42	6
1,6	25	3
1,8,12	11	1.5
1,7,12	11	1.5
1,6,12	11	1.5
1,7,11	19	2
1,6,11	19	2
1,6,10	4	.5

Figure 12. Career paths, bottleneck flows and bottleneck flow percentages